International Baccalaureate®
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MATHEMATICS SL TZ1

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 42	43 - 55	56 - 68	69 - 80	81 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2009 examination session the IB has produced time zone variants of the Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

Important information

Teachers are advised that the set of tasks published by the IB for 2009-10 are for those sessions only and should not be used for submission as part of the portfolio after the November 2010 session. They can be used for practice purposes and teachers are encouraged to provide such practice opportunities where time allows. A new set of tasks is in development for 2011-2012. Consult the DP Coordinator Notes and the Online Curriculum Centre (OCC) for updates on the distribution of these tasks. The IB continues to encourage teachers to design and use tasks that fit their own circumstances, providing that these tasks allow the opportunity for students to achieve success at all levels of all criteria.

It is essential for schools to include background information, copies of the tasks, solutions/marking keys and teacher comments with samples submitted. All of these will be very useful in establishing the reasons for the achievement levels awarded by the teacher. It will be even more helpful if teacher expectations on how criteria levels are awarded can be provided in a matrix format, similar to the published overview of assessment criteria, or the matrices in the resources section of the OCC. For schools with more than one teacher for Mathematics SL, such a matrix can serve to help internal standardisation, to ensure consistent marking within the schools.

The range and suitability of the work submitted

This was the first session where teachers were allowed to use the new tasks published for 2009/2010. A 10-mark penalty was applied for using old tasks from the Teacher Support Material documents (TSM). As a result, almost all schools had selected appropriate ones from the published tasks. The most popular Type I task appears to be "Matrix Binomials", with "Logarithm Bases" the next favourite. For the Type II task, "Body Mass Index" and "Crows Dropping Nuts" were most widely used whereas "Logan's Logo" was used rarely. Naturally, these would all meet the Internal Assessment requirements without any problems.

School-designed tasks still varied from suitable to unacceptable. Investigations that were overly prescriptive precluded students from achieving higher levels on Criteria C and D. There were also recycled old tasks from the previous course which were not sufficiently modified. Teachers are reminded that before they assign a task to students, they should work through it, to ensure that it matches the assessment criteria well and can allow for candidates to achieve maximum performance. An incomplete portfolio should not be submitted as part of a sample. If selected, it should be accompanied by another portfolio of approximately the same mark.

Candidate performance against each criterion

Criterion A: This remained the easiest for students to attain the highest level of 2. They were nearly all aware of the proper terminology for the required topics and were able to use the corresponding notation consistently. Still, some of them did not recognize the importance of using meaningful variables for different model functions in modelling tasks. Others had repeatedly used calculator/computer notation, like "^", "*", [A], which were not penalised by their teachers. The correct use of the "approximately equal" sign should also be enforced, especially in a modelling task, due to the approximate nature of the context.

Criterion B: Most candidates also did well on this criterion and were properly assessed by most teachers. By using appropriate graphs and tables, they would normally achieve levels of 2 or higher. However, a lack of introduction plus poor commentary, a purely “question and answer” format, or student work that requires constant reference to the task statement would lead to a penalty of one mark. The tasks require mathematical writing, not a response to a set of homework exercises.

Type I Criteria C and D: Most students were successful in identifying the patterns and generalizing the results, thus reaching a level 3 in Criterion C, even though in some cases, general statements would appear out of the blue without any analysis or development. A level 4 could then be achieved with a correct and successful mathematical analysis, even if it was not the expected general statement. This could go further up to C5 provided that the general statement was tested against further examples (note the plural!) and/or informally justified. Note that validation of a general statement requires comparing the statement and its results to the actual mathematical behaviour that is the basis of the investigation. Simply substituting new values into a statement and obtaining a result does not verify that the statement reflects the pattern.

Similarly, these students would have no difficulty obtaining a mark of 3 for Criterion D. However, any higher marks proved to be hard to attain since testing for scope and limitations, linked with verification, remained a student weakness. Most students considered only positive integral values, ignoring the possibility of real values for variables. Given the access to technology, it is expected that students will try negatives, fractions, decimals, radicals, etc in their statement as the situation allows. As a result, it was quite common for a D score to be moderated down to a 3 or 4, subject to the presentation of informal justification.

Type II, Criteria C and D: Most candidates did a good job in deriving the suitable models analytically, and then, considering how well the models fit the data, at least, qualitatively. From time to time, some teachers did not regard this to be sufficient for awarding a level 4 in Criterion C, yet a good qualitative response is all that is expected for mathematics SL. Unfortunately, there were still cases in which models were entirely developed by using regression tools, without going through any analytical analysis. This approach allows a maximum mark of C2. To reach the highest level of 5, there must be evidence of applying the student-developed model to further data or another situation. Most candidates also achieved 2 or 3 for Criterion D, indicating an attempt to interpret the reasonableness of the results. However, to be able to make meaningful comments relating to the task or within the context continued to be challenging for them.

Criterion E: The use of technology varied from routine calculations to a full and resourceful application. A lack of background information on available technology makes it difficult to confirm the teacher’s assessment. There must be clear evidence on the use of technology within student work to support the highest level of 3 for Criterion E. Quite often teachers' expectations were not consistently applied within the same school samples.

Criterion F: In spite of the fact that F2 was sometimes awarded too casually (even for portfolios with missing parts), Criterion F was generally well understood, with a high level of confirmation. This was mostly due to the fact that the majority of students were given the satisfactory level of 1, as would be expected.

Recommendation and guidance for the teaching of future candidates

Use of tasks

Teachers should feel free to adapt, modify and rewrite the published tasks to best suit their own students due to their diverse background and international nature. They should ensure at the same time that the resulting version provides for full success against the criteria. It would be a good idea to share teacher-designed, non-TSM tasks on the OCC first for advice before actually assigning them to the students.

Specific practice

A variety of tasks, especially for Type II, should be provided. Teachers should develop the process of searching for patterns focussing on the analysis of the data for an investigation task. For the modelling tasks, they should lead students through the development of a mathematical model starting with defining variables, specifying parameters and identifying constraints. Evaluation of results within context is another skill that requires nurturing. Students should also be shown examples of good portfolios, like using correct notation or finding a modelling function algebraically by solving a system of n equations with n unknowns. Some discussion in class regarding the actual context can help focus the students' interpretations of the scenario.

Comments on the work

Teacher annotations and markings, with precise and specific comments on the students' work serve to facilitate the moderation. They make it easier for the moderators to follow the reasoning behind the teachers' assessment, and also for the students to learn from their own mistakes. Generic feedback such as "good" or "consistent" should be supported with details. Simply rephrasing the criterion level descriptors is not helpful. It is important that all comments are legible. Please take the time to make your comments readable to both the student and the moderator.

Guidance for students

All students should be given copies of a full description of portfolio instructions and the assessment criteria to facilitate their understanding of the entire procedure and setting of their own targets on this internal component. In particular, when a new task is given, they should be clearly informed of the expectations of each assessment criterion.

Technology

Teachers should explore and discuss with students what constitutes resourceful use of technology. Using graphs to reflect or reinforce numerical patterns, using spreadsheets to produce or confirm results for large values of the variable(s), showing an evolution of development of a model through an ever-improving sequence of graphical transformations, or comparing multiple scenarios on the same set of axes to clearly show similarities and differences are some ways in which the effective use of technology can be demonstrated.

External assessment

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 23	24 - 35	36 - 47	48 - 60	61 - 72	73 - 90

The areas of the programme which proved difficult for candidates

Candidates showed difficulty answering questions pertaining to:

- Conditional probability
- Gradient of a normal to a curve
- Transformations of graphs of functions
- Domain of an inverse function

- Properties of logarithms
- Use of the second derivative to justify a maximum
- Properties of definite integrals

The levels of knowledge, understanding and skill demonstrated

Where questions focused on procedural knowledge of a topic, candidates demonstrated adequate skill. Questions that required a more conceptual understanding proved more challenging. At times candidates were unable to interpret the mathematical language of the situation (see comments for Q2, Q6 and Q8).

Candidate strengths included the following areas:

- Matrix algebra
- Simple derivatives
- Finding an inverse function
- Volume of revolution
- Vector calculations

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1 (matrix algebra)

Matrix multiplication was generally well done. In writing down the inverse matrix, few made the connection that $AB = 4I$ implies $A^{-1} = \frac{1}{4}B$, as most used the formula in the booklet. Many proceeded to solve the matrix equation successfully, either by setting up a system of two equations or using $A^{-1}C$. Of those who used the inverse, some reversed the order of multiplication, incorrectly multiplying CA^{-1}.

Question 2 (conditional probability)

Most candidates answered part (a) correctly. Few candidates used the concept of "B given A" to simply "write down" the answer of $\frac{2}{10}$. Instead, most reached for the formula in the booklet, with which few were successful. Few also made the connection that part (c) could be answered using both previous answers. Many found $\mathrm{P}\ A \cap B$ correctly even when answering part (b) incorrectly, although some candidates did not decrease the denominator for the second event.

Question 3 (gradient of normal to a curve)

Candidates familiar with the product rule easily found the correct derivative function. Many substituted π to find the tangent gradient, but surprisingly few candidates correctly considered that the gradient of the normal is the negative reciprocal of this answer.

Question 4 (kinematics)

Many candidates answered this question completely and correctly, showing a good understanding of the graphical relationship between displacement, velocity and acceleration. When done incorrectly, many answered with the displacement as graph B and acceleration as graph C. Many of these candidates found the value of t which gave a maximum in the remaining graph, and were awarded follow through marks.

Question 5 (function transformations)

The translation was often described well as horizontal (or shift) one unit right. There was considerable difficulty describing the vertical stretch as it was often referred to as "stretch by 2" or "amplitude of 2". A full description should include the name (e.g. vertical stretch) and value for full marks. Candidates also had difficulty applying two consecutive transformations to a single point. Often the translations were applied directly to $(-1,1)$ instead of first mapping from f to g. A good number of candidates correctly found $h(x)$, but most could not find P from this function.

Question 6 (inverse functions, log properties)

Many candidates interchanged the x and y to find the inverse function, but very few could write down the correct domain of the inverse, often giving $x \geq 0$, $x > 3$ and "all real numbers" as responses. Where students attempted to solve the equation in (b), most treated

$\ln x - 3$ as $\ln(x-3)$ and created an incorrect equation from the outset. The few who applied laws of logarithms often carried the algebra through to completion.

Question 7 (volume of revolution)

Despite the “reverse” nature of this question, many candidates performed well with the integration. Some forgot to square the function, while others did not discard the negative value of a. Some attempted to equate 32π to the formula for volume of a sphere, which suggests this topic was not fully covered in some centres.

Question 8 (trigonometric ratios, double-angle identity, chain rule, justifying a maximum)

Candidates familiar with the circular nature of sine and cosine found part (a) accessible. However, a good number of candidates left this part blank, which suggests that there was difficulty interpreting the meaning of the x and y in the diagram.

Those with answers from (a) could begin part (b), but many worked backwards and thus earned no marks. In a “show that” question, a solution cannot begin with the answer given. The area of the rectangle could be found by using $2x \times 2y$, or by using the eight small triangles, but it was essential that the substitution of the double-angle formula was shown before writing the given answer.

As the area function was given in part (b), many candidates correctly found the derivative in (c) and knew to set this derivative to zero for a maximum value. Many gave answers in degrees, however, despite the given domain in radians.

Although some candidates found the second derivative function correctly, few stated that the second derivative must be negative at a maximum value. Simply calculating a negative value is not sufficient for a justification.

Question 9 (vectors)

Combining the vectors in (a) was generally well done, although some candidates reversed the subtraction, while others calculated the magnitudes.

Many candidates successfully used scalar product and magnitude calculations to complete part (b). Alternatively, some used the cosine rule, and often achieved correct results. Some assumed the triangle was a right-angled triangle and thus did not earn full marks. Although

PQR is indeed right-angled, in a "show that" question this attribute must be directly established.

Many candidates attained the value for sine in (c) with little difficulty, some using the Pythagorean identity, while others knew the side relationships in a 30-60-90 triangle. Unfortunately, a good number of candidates then used the side values of $1, 2, \sqrt{3}$ to find the area of PQR, instead of the magnitudes of the vectors found in (a). Furthermore, the "hence" command was sometimes neglected as the value of sine was expected to be used in the approach.

Question 10 (points of inflexion, area and integration, log properties, function transformations)

Part (a) was achieved by some candidates, although brackets around the $-x$ were commonly neglected. Some attempted to show the relationship by substituting a specific value for x. This earned no marks as a general argument is required.

Although many recognized the requirement to set the second derivative to zero in (b), a majority neglected to give their answers as ordered pairs, only writing the x-coordinates. Some did not consider the negative root.

For those who found a correct expression in (c)(i), many finished by calculating $\ln 50 - \ln 10 = \ln 40$. Few recognized that the translation did not change the area, although some factored the 2 from the integrand, appreciating that the area is double that in (c)(i).

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 50	51 - 61	62 - 72	73 - 90

The areas of the programme which proved difficult for candidates

The following areas provided difficulties for candidates:

- Data handling and statistics
- Entering statistical data into a calculator to find the standard deviation

- Using inverse normal probabilities to find a value
- Period and range of more complex trigonometric functions
- Conditional probability
- Matrices and vectors
- Applying first principles to find the derivative of a function
- Writing answers to 3 significant figures and rounding numbers correctly.

The levels of knowledge, understanding and skill demonstrated

It was pleasing to see a large number of candidates demonstrate a comprehensive knowledge and understanding of the syllabus. For the most part, candidates demonstrated a good balance between using the GDC and solving problems algebraically. In general, candidates are doing a much better job of showing their work with less GDC notation being used.

The area that caused the most trouble for candidates were questions associated with probability or statistics. It is obvious that many candidates fail to understand the underlying principles of the different statistical processes that they are expected to carry out. Candidates were often confused as to when and how to use the normal distribution or the binomial distribution and often applied them to any probability distribution.

Many candidates did very well on vectors, geometric and arithmetic progressions and calculus, but continue to have difficulties knowing when and how to use their GDCs appropriately to solve problems. Those candidates demonstrating skill in using both analytical and geometrical techniques had little difficulty with this paper.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1 (Arithmetic series)

This question was generally well answered. Many candidates who got part (a) wrong, recovered and received full follow through marks in part (b). There were a few candidates who confused the term and sum formula in part (a).

Question 2 (Circle trigonometry)

This question was well answered by the majority of candidates. Full solutions were common in both parts, and a variety of successful approaches were used. Radians were well handled with few candidates working with the angle in degrees. Some candidates incorrectly found the length of the arc subtended by the central angle rather than the length of segment [AB]. In part (b), some candidates incorrectly subtracted the area of the triangle or even a length. Many candidates failed to give answers to 3 significant figures and therefore lost an accuracy mark.

Question 3 (Functions)

The majority of candidates handled the composition of the two given functions well. However, a large number of candidates had difficulties simplifying the result correctly. The period and range of the resulting trig function was not handled well. If candidates knew the definition of "range", they often did not express it correctly. Many candidates correctly used their GDCs to find the period and range, but this approach was not the most efficient.

Question 4 (Normal distribution)

It remains very clear that some centres still do not give appropriate attention to the normal distribution. This is a major cause for concern. Most candidates had been taught the topic but many had difficulty understanding the difference between $z, F(z), a$ and x. Very little working was shown which demonstrated understanding. Although the GDC was used extensively, candidates often worked with the wrong tail and did not write their answers correct to 3 significant figures.

Many candidates had trouble with part (b), a majority never found the complement, instead using their GDCs to calculate the result, which many times was finding a for $P(X \le a) = 0.27$ instead of for $P(X \ge a) = 0.27$. Many others substituted the values of 0.27 or 0.73 into the equation, instead of the z-scores.

Question 5 (Intersection of lines)

If this topic had been taught well then the candidates scored highly. The question was either well answered or not at all. Many candidates did not understand what was needed and tried to find the length of vectors or mid-points of lines. The other most common mistake was to use the values of the parameters to write the coordinates as $\mathrm{P}(2,-1)$.

Question 6 (Geometric series)

Part (a) was well done. In part (b) a good number of candidates did not realize that they could use logs to solve the problem, nor did they make good use of their GDCs. Some students did use a trial and error approach to check various values however, in many cases, they only checked one of the" crossover" values. Most candidates had difficulty with notation, opting to set up an equation rather than an inequality.

Question 7 (Probability)

Part (a) was nearly always correctly answered by those who attempted the question, but part (b) (conditional probability) was poorly done. A surprisingly small number of students drew a tree diagram in part (a) and those who did answered this part and part (b) well. Many found the correct complement in part (b) but could not make any further progress.

Question 8 (Cumulative frequency, probability)

Part (a) defeated the vast majority of candidates who clearly had not been taught data entry. Some schools had attempted to teach how to use a formula rather than the GDC to find the standard deviation and their students invariably used this formula incorrectly. Use of the GDC was not only expected but should be emphasised as stated in the syllabus.

Part (b) revealed poor understanding of cumulative frequency and the IQR was often reported as an interval. The remaining parts of this question were generally answered well although a number of candidates had difficulty with using the formula for expected value.

Question 9 (Equations of curves, matrices)

There was wide spectrum of success on this problem. Candidates who were comfortable with their GDC had little difficulty obtaining full marks. Parts (a), (b) and (c) (i) were well answered, although the transpose of $\boldsymbol{A}$, instead of $\boldsymbol{A}$ was a common mistake. Most candidates were able to find the inverse of **their** matrix $\boldsymbol{A}$ but often seemed to transcribe the information from their calculator incorrectly.

The majority of candidates were able to use matrix methods to complete part (c) (iii) but some attempted to set up a system of equations, which usually led to algebraic errors. Part (d) was designed for candidates to use their GDC to find the vertex, but again, many attempted to complete the square and a variety of algebraic errors were made.

Question 10 (Calculus, gradients and tangents)

In part (a), the basic expansion was not done well. Rather than use the binomial theorem, many candidates opted to expand by multiplication which resulted in algebraic errors. In part (b), it was clear that many candidates had difficulty with differentiation from first principles. Those that successfully set the answer up, often got lost in the simplification.

Part (c) was poorly done with many candidates assuming that the tangents were horizontal and then incorrectly estimating the maximum of f as the required point. Many candidates unnecessarily found the equation of the tangent and could not make any further progress. In part (d) many correct solutions were seen but only a very few earned the reasoning mark. Part (e) was often not attempted and if it was, candidates were not clear on what was expected.

The type of assistance and guidance the teachers should provide for future candidates

It may be helpful to emphasize concepts over formulas and procedures when preparing students for this examination. Formulas can be helpful mathematical tools, and procedures are necessary skills, but they do not replace understanding of mathematical principles. For example, knowledge of the concept of the probability of A given B can make such a question answerable by simple counting, but most candidates tend to mire themselves in the often unnecessary and sometimes slippery calculations of the formula. The question on kinematics on paper one assessed graphical relationships between concepts of motion rather than ability to find a first or second derivative function.

Although a variety of methods are often acceptable, teaching students to choose more efficient strategies can help to save time and possibly avoid arithmetical errors. For example, in solving the matrix equation in the first question on paper one, students were expected to use the inverse matrix, but many chose to create and solve a separate system of simultaneous equations. This earned full marks when done correctly, but much time is forfeited with this approach.

Candidates should be instructed to consider both geometric and analytical approaches to solving problems to facilitate understanding. Many candidates have difficulty with the underlying concepts as they are only aware of which buttons to press on their calculators. As such, they are unable to apply their knowledge to a problem presented in a slightly different way. When preparing candidates for future examinations, emphasizing a graphical understanding in conjunction with analytical techniques is essential.

It is noticeable that successful candidates have work that is clearly set out while unsuccessful candidates usually display an incoherent structure to their work. An inability to organize one's mathematical thoughts is a hindrance in a timed examination. Emphasizing high quality mathematical communication can help students learn to organize their thoughts and become more efficient mathematical thinkers.

Students continue to struggle with the expectations of a "show that" question. Thinking backwards can sometimes be a helpful mental strategy, but the written work must show a deductive path from some mathematical principle that clearly leads to the desired result, without any backwards thinking from the answer given.

Teachers should not only spend more time on the normal distribution but also stress the need for correct notation and the usefulness of drawing a diagram. Most candidates appeared to have some familiarity with probability, the Normal Distribution and the Binomial Distribution but many were confused about how and when to use them. More time investigating the properties and usefulness of each of these distributions would be time well spent.

Candidates should be instructed to complete their answers and not, for example, leave answers with decimals inside fractions. Similarly, answers left in the form $\begin{pmatrix} n \\ r \end{pmatrix}$ are also considered incomplete.

Teachers should continue to stress the meaning of the command terms and have students look at the number of marks allocated to each question part to determine how much "work" they should show.

Teachers should continue to work with students on writing their answers to the correct number of significant figures.

Having said, many teachers are doing an excellent job in preparing their students for the examinations, and should continue with the good work.